# Global Journal of Engineering Science and Researches BUTTERFLY CURVE BASED BIOGEOGRAPHY BASED OPTIMIZATION ALGORITHM <br> Ruchi Mishra ${ }^{* 1}$ \& Neha Mishra ${ }^{2}$ <br> ${ }^{*}, 2$ Rajasthan Technical University, Department of Computer Science and Engineering, Kota, India 


#### Abstract

Biogeography based optimization (BBO) is a population-based evolutionary optimization algorithm inspired by the science of biogeography. To enhance the convergence rate of the algorithm towards the optimal solution a new strategy is introduced named as a butterfly curve based BBO (BFBBO) algorithm. In this strategy, a new phase is introduced in which the rotated butterfly curve equation is incorporated to balance the step size. This proposed algorithm is also tested over 20 benchmark problems. The results are also compared with BBO, gbest inspired biogeography based optimization (GBBO), and particle swarm optimization (PSO). The outcome reveals the competence of the proposed algorithm in the area of evolutionary-based algorithms.


Keywords: Biogeography based optimization; Butterfly curve; Optimization.

## I. INTRODUCTION

Nature Inspired Algorithms (NIAs) [1] are an important source for motivating intelligent systems. It also gives solutions for difficult optimization problems. A function is optimized by evolutionary algorithms by generating a candidate solution in terms of the measure of goodness. The essential characteristics of biogeography are maintained by Biogeography based optimization (BBO) [2]. Biological species can be split according to space and time by researching biogeography. In BBO solution features are migrated between species is motivated by the scientific model of biogeography. Research for improving the performance of BBO is going on. It is reported in the literature that BBO has stagnation [3] problem.

In the above context, this article proposes an efficient BBO algorithm. This phase balances the step size of the solutions. The algorithm is titled as butterfly curve based BBO algorithm (BFBBO). This proposed phase is also tested over 20 test problems and the results are also compared with BBO [2], gbest inspired biogeography based optimization (GBBO) [4] and particle swarm optimization (PSO) [5]. The obtained outcome prove the authenticity of the discovered approach.

The other sections are organized as: In Section 2, BBO is discussed. In Section 3, we introduced butterfly curve inspired local search strategy and also discovered BFBBO algorithm in Section 4. For measuring the performance of BFBBO algorithm a comparison has been made with various algorithms in Section 4. At last, Section 5 includes the conclusion of the proposed work.

## II. OVERVIEW OF BBO ALGORITHM

BBO [2] algorithm is population-based algorithm. It is modeled on the science of biogeography. The model of biogeography explains Migration of species (Birds, animals), Speciation (Development of species) between habitat (island) and species Extinction. In BBO [2] habitat (Island) suitability Index is analogs to an island which is considered as an individuals. Habitat (solution) is known with high HSI are devoted to life. HSI corresponds to the BBO solution's goodness. Rainfall, topographic diversity, temperature, land area, vegetation diversity, and others are some aspects which are included with HSI. Suitability index variable (SIVs) are known as aspects which identify habitability. Figure 1 demonstrate a species prolific's model in a single type of island.


Fig. 1: Species Model [2]
Habitability is a term in which SIVs is the island's independent variable and the dependent variable is HSI. High-HSI habitats are the habitat with a large number of species and Low habitats are habitats with few species. Facets of High HSI solution is given to low HSI, these facets are acquired by Low HSI provided by High HSI solution. Facets of High HSI solution is to emigrate to Low HSI solution. Emigration and Immigration tends to reform the solutions and thus emerging a solution to optimization problem. Like other evolutionary population dependent algorithms, BBO solution search procedure is an iterative procedure. After, BBO population's initialization migration and mutation are the two type of procedures which necessitates the recited iterations. The schemes are described as.

## 1. Initialisation of the population

The arbitrarily depressed population of habitat is created by BBO where every habitat Hi ( $\mathrm{i}=1,2, \ldots$, population) is a d-dimensional vector (number of variables). In this procedure, Hi represents the ith solution in the population. Every solution is created using the following eq. 1:
$\mathrm{Hi} j=\operatorname{Hmin} \mathrm{j}+\operatorname{rand}[0,1](H \max \mathrm{j}-\operatorname{Hmin} \mathrm{j})$
Here Hmin j and Hmax j are limits of Hi in jth dimension and rand $[0,1] \varepsilon(0,1)$.

## 2. Migration

By taking advantage of the emigration rate $(\mu \mathrm{j})$ as well as immigration rate ( $\lambda \mathrm{i}$ ) facets are probabilistically shared between the habitats this procedure is known as Migration [6]. To bestowing the facets between candidates solution for modifying goodness the migration operator is liable. According to the probability of $(\mu \mathrm{j})$ and $(\lambda \mathrm{i})$ emigration solution and immigrtion solutions are selected respectivliy. Immigrating solution (Hi) SIV is interchanged by emigrating solution's (Hj) SIV when the decision is made about which solution feature (SIV) of the immigrating solution (Hi) is required to be modified.
new $($ Hik $)=$ Hik $+\alpha(H j k-H i k)$
here $\alpha$ is the user defined migration operator and k is the dimension of the solution.

## 3. Mutation

In BBO algorithm solution's variety are keeping the mutation [7] is culpable. For low and high HSI candidate solutions mutation renders a possibility for improving the solution's goodness. It is able to intensify the solution's quality even if they have more innumerable solutions already.

## 4. Pseudo-code of the BBO algorithm

From the above discussion in section 2, BBO's pseudo-code is depicted in algorithm 4.

```
Algorithm 1 BBO Algorithm
Generate a uniform random set-of-solutions \(H 1, H 2, \ldots, H n\);
Each solution's goodness (HSI) has been calculated;
while the termination criteria met do
From best to worst solutions has been sorted;
Based on HSI for every solutions \(\lambda\) and \(\mu\) has been calculated;
Procedure of migration has been applied;
For each solutions probability has been renewd;
Procedure of mutation has been applied;
Every solution's goodness has been calculated;
For maintaining best solutions elitism concept has been applied;
end while
```


## III. BUTTERFLY CURVE INSPIRED LOCAL SEARCH STRATEGY

With large scaly wings, butterflies are beautiful, flying insects. Like all other insects, butterflies have an exoskeleton, compound eyes, a pair of antennae, 3 body parts and six jointed legs. Head, thorax (the chest), and abdomen (the tail end) are the 3 body parts of a butterfly. By tiny sensory hairs, the body is covered of a butterfly. Thorax is connected with four wings and the six legs of the butterfly. Legs and wings of butterfly move with the help of muscles which containing by thorax. Butterflies are said to be very good fliers because they have two pair of colorful large wings. Butterfly's thorax (mid-section) attached with the wings. Delicate wings are supported by veins and also nourish wings by veins with blood. Above 86 degrees body temperature is required for the butterfly to fly. In cool weather, butterflies sun themselves to warm up. The color of the wings fades and become ragged as butterflies age. Among butterfly species speed has been varied. About 30 miles per hour or faster is the fastest speed of butterfly's fly. About 5 mph , butterflies fly with slow flying.


Fig. 2: Butterfly line curve

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At sea level, high in the mountains, cold, dry, hot and moist environments, butterflies are found all over the world. In tropical rainforests area, a butterfly is mostly found. Butterfly migration is not well conjectured.
$\mathrm{R}=\mathrm{e} \sin \theta-2 \cos 4 \theta+\sin 5(\theta-(\Pi / 2) / 12)$
Butterfly curve can be written as eq. 3. The horizontal distance R that a butterfly travels is related to the take-off angle $\theta$ at take-off. Where $\theta$ is usual polar co-ordinates. The key point is that if a butterfly takes off at this optimal angle of $60^{\circ}$. The proposed local search strategy named as butterfly curve inspired local search strategy (BFLS).
Based upon the eq. 3 this article proposes an efficient local search strategy. During the local search process the best solution of the swarm is updated its position using the following equation:
xbjest $\mathrm{j}=\mathrm{xbest} \mathrm{j}+\mathrm{R} * \mathrm{l}$
(4)

Where, xbjest j is the updated position of the best solution of the swarm and 1 presents the balancing factor as shown below:

$$
\begin{equation*}
1=5-5 * \theta / 360 \tag{5}
\end{equation*}
$$

The value of $\theta$ varies from $0^{\circ}$ to $360^{\circ}$ degree. The value of $\theta$ is calculated as per the equation 6 .
Here $t$ represents the current iteration of the local search. The total number of local search iteration $T$ is decided based upon an extensive analysis which is mentioned in the experimental setting. The pseudo-code of the proposed local search strategy BFLS. In the hope to diminish the algorithm's step size optimization algorithms are hybridized with local search strategies. For reducing the algorithm's step size the rotated butterfly curve is incorporated with the BBO algorithm in the intended article. The intended algorithm is known as the butterfly curve inspired BBO algorithm. Intended algorithm's pseudo-code is as follows:
It is clear from the Algorithm 4 that the BFLS strategy is incorporated after the elitism concept of the BBO algorithm. Therefore, in the proposed BFBBO algorithm, the best solution found after executing steps of BBO algorithm is given more chances to search in the vicinity with small step sizes to exploit the nearby area using the BFLS strategy. This will improve the exploitation capability of the BBO algorithm.

After the BBO algorithm's mutation, the rotated butterfly curve is incorporated that is very much clear from the algorithm 4. Therefore, in the intended BFBBO algorithm, after executing all phases the best solution has been found. By using the curve for searching in the vicinity with small step sizes to exploit the nearby area more chances have been given. Further, the incorporation of the curve also improves the BBO algorithm's convergence ability which makes, the intended BFBBO, a cost-effective algorithm in terms of numerous function evaluations.

## Algorithm 2 Butterfly Curve Inspired Local Search Strategy (BFLS)

Input optimization function $\operatorname{Min} f(x)$;
Select the best solution $x_{\text {best }}$ in the swarm which is going to modify its position;

Initialize iteration counter $=0$ and total iterations of BFLS, T;
while ( $\mathrm{t}<\mathrm{T}$ ) do
Generate a new solution $x_{\text {best }}$ using in Algorithm 3;
Calculate the objective value $\mathrm{f}\left(x^{\prime}{ }_{\text {best }}\right)$;
if $x_{\text {best }}^{\prime}<x_{\text {best }}$
$x_{\text {best }} x^{\prime}{ }_{\text {best; }}$
end if
$\mathrm{t}=\mathrm{t}+1$
end while

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Algorithm 3 New solution generation

```
Input best solution }\mp@subsup{x}{\mathrm{ best }}{}\mathrm{ from the population;
Randomly select a solution }\mp@subsup{x}{i}{}\mathrm{ from the population;
Initialize the value of 0=60*t /*t is the current iteration counter */;
for }\textrm{j}=1\mathrm{ to D do
if }\textrm{U}(0,1)<\mp@subsup{C}{r}{}/*\mp@subsup{C}{r}{}\mathrm{ is the perturbation rate, a constant in the range
(0,1)*/
x'best j=xbest j;
else
x'best j= xbest j+R*l;
end if
end for
Return x'best j
```

Algorithm 4 Butterfly Curve Inspired Biogeography Based Algorithm (BFBBO)
Generate a uniform random set-of-solutions $H 1, H 2, \ldots, H n$;
Each solution's goodness (HSI) has been calculated;
while the termination criteria met do
From best to worst solutions has been sorted;
Based on HSI for every solutions $\lambda$ and $\mu$ has been calculated;
Procedure of migration has been applied;
For each solutions probability has been renewd;
Procedure of mutation has been applied;
Every solution's goodness has been Calculated;
For maintaining best solutions elitism concept has been applied;
apply butterfly curve inspired phase;
end while
best solution has been printed


Fig. 3: Graph showing exploitation ability(reducing step size)

## IV. RESULT

## 1. Test Problems

For analyzinng the functioning of the intended BFBBO algorithm, 20 distinct globally occupied optimizationfunctions ( $\mathrm{f} u \mathrm{uc} 1$ to $\mathrm{f} u n \mathrm{c} 20$ ) are picked as shown in Table 1.

Table 1. Test-problems: TPs, D: Dimension, AE: Acceptable-Error

| S-NO. | Test Problems | Objective Function | Search Range | D | AE |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Sphere | func $_{1}(\mathrm{x})=\sum_{i=1}^{D} x_{i}^{2}$ | [-5.12, 5.12] | 10 | $1.0 \mathrm{E}-05$ |
| 2 | De Jong f4 | func $_{2}(\mathrm{x})=\sum_{i=1}^{D} i . x_{i}^{4}$ | [-5.12, 5.12] | 30 | $1.0 \mathrm{E}-05$ |
| 3 | Griewank | $\begin{aligned} & \text { func }_{3}(\mathrm{x})=\quad 1+\frac{1}{4000} \sum_{i=1}^{D} x_{i}^{2}- \\ & \prod_{i=1}^{D} \cos \left(\frac{x_{i}}{\sqrt{i}}\right) \end{aligned}$ | [-600, 600] | 30 | $1.0 \mathrm{E}-05$ |
| 4 | Rosenbrock | $\begin{aligned} & \operatorname{func}_{4}(\mathrm{x})=\sum_{i=1}^{D}\left(1 0 0 \left(x_{i+1}-\right.\right. \\ & \left.\left.x_{i}^{2}\right)^{2}+\left(x_{i}-1\right)^{2}\right) \end{aligned}$ | [-30, 30] | 30 | $1.0 \mathrm{E}-02$ |
| 5 | Rastrigin | $\begin{aligned} & \operatorname{func}_{5}(\mathrm{x})= \\ & \left.10 \cos \left(2 \pi x_{i}\right)\right] \end{aligned} \quad 10 \mathrm{D}+\sum_{i=1}^{D}\left[x_{i}^{2}-\right.$ | [-5.12, 5.12] | 30 | $1.0 \mathrm{E}-03$ |
| 6 | Ackley | $\begin{aligned} & \operatorname{func}_{6}(\mathrm{x})= \\ & 20+\mathrm{e}+\exp \left(-\frac{0.2}{D} \sqrt{\sum_{i=1}^{D} x_{i}^{3}}\right) \end{aligned}$ | $[-1,1]$ | 30 | $1.0 \mathrm{E}-05$ |
| 7 | Alpine | $\begin{aligned} & \operatorname{func}_{7}(\mathrm{x})= \\ & \left.0.1 x_{i} \mid\right) \end{aligned}$ | $[-10,10]$ | 30 | $1.0 \mathrm{E}-05$ |
| 8 | Michalewicz | $\begin{aligned} & \operatorname{func}_{8}(\mathrm{x})= \\ & \sum_{i=1}^{D} \sin x_{i}\left(\sin \left(i \frac{x_{i}^{2}}{\pi}\right) 20\right) \end{aligned}$ | $[0, \pi]$ | 10 | $1.0 \mathrm{E}-05$ |
| 9 | Cosine Mixture | $\begin{aligned} & \text { func }_{9}(\mathrm{x})= \\ & 0.1\left(\sum_{i=1}^{D} \cos 5 \pi x_{i}\right)+0.1 D \end{aligned}$ | $[-1,1]$ | 30 | $1.0 \mathrm{E}-05$ |
| 10 | Exponential | func $_{10}(\mathrm{x})=-\left(\exp \left(-0.5 \sum_{i=1}^{D} x_{i}^{2}\right)\right)+1$ | $[-1,1]$ | 30 | $1.0 \mathrm{E}-05$ |
| 11 | Zakharov | $\begin{aligned} & \text { func }_{11}(\mathrm{x})= \\ & \left(\sum_{i=1}^{D} \frac{i x i}{2}\right)^{2}+\left(\sum_{i=1}^{D} \frac{i x 1}{2}\right)^{4} \sum_{i=1}^{D} x_{i}^{2}+ \end{aligned}$ | [-5.12, 5.12] | 30 | $1.0 \mathrm{E}-02$ |
| 12 | Cigar | func $_{12}(\mathrm{x})=x_{0}^{2}+100000 \sum_{i=1}^{n} x_{i}^{2}$ | [-1010] | 30 | $1.0 \mathrm{E}-05$ |
| 13 | brown3 | $\begin{aligned} & \operatorname{func}_{13}(\mathrm{x})=\quad \sum_{i=1}^{D-1}\left(x_{i}^{2\left(x_{i}+1\right)^{2}+1}+\right. \\ & \left.x_{i+1} 2 x_{i}^{2}+1\right) \end{aligned}$ | [-1, 4] | 30 | $1.0 \mathrm{E}-05$ |
| 14 | Schewel prob 3 | func $_{14}(\mathrm{x})=\sum_{i=1}^{D}\left\|x_{i}\right\|+\prod_{i=1}^{D}\left\|x_{i}\right\|$ | [-10, 10] | 30 | $1.0 \mathrm{E}-05$ |
| 15 | Salomon Problem (SAL) | $\begin{aligned} & \operatorname{func}_{15}(\mathrm{x})= \\ & \cos \left(2 \pi \sqrt{\sum_{i=1}^{n} x_{i}^{2}}+0.1\left(\sqrt{\sum_{i=1}^{n} x_{i}^{2}}\right)\right. \end{aligned}$ | $\left[\begin{array}{lll} -100 & 100] \end{array}\right.$ | 30 | $2.0 \mathrm{E}-01$ |
| 16 | Axis parallel hypere llipsoid | $\operatorname{func}_{16}(\mathrm{x})=\sum_{i=1}^{D} i x_{i}^{2}$ | [-5.12, 5.12] | 30 | $1.0 \mathrm{E}-05$ |
| 17 | Pathological Function | $\begin{aligned} & \operatorname{func}_{17}(\mathrm{x})=\quad \sum_{i=1}^{D-1}(0.5+ \\ & \left.\frac{\sin ^{2} \sqrt{\left(100 x_{i}^{2}+x_{i+1}^{2}\right)-0.5}}{1+0.001\left(x_{i}^{2}-2 x_{i} x_{i+1}+x_{i+1}\right)^{2}}\right) \end{aligned}$ | [-1,1] | 30 | $1.0 \mathrm{E}-01$ |
| 18 | Sum of different powers | func $_{18}(\mathrm{x})=\sum_{i=1}^{D}\left\|x_{i}\right\|^{i+1}$ | $[-1,1]$ | 30 | $1.0 \mathrm{E}-05$ |
| 19 | Step function | func $_{19}(\mathrm{x})=\sum_{i=1}^{D}\left(\left\|x_{i}+0.5\right\|\right)^{2}$ | [-100, 100] | 30 | $1.0 \mathrm{E}-05$ |
| 20 | Rotated hyper-ellipsoid | func $_{20}(\mathrm{x})=\sum_{i=1}^{D} \sum_{j=1}^{i} x_{j}^{2}$ | $\begin{aligned} & {[-65.536,} \\ & 65.536] \end{aligned}$ | 30 | $1.0 \mathrm{E}-05$ |

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## 2. Parameter Setting

For testing the performance of BFBBO, comparative-analysis is carried out among BFBBO, BBO [2], GBBO [4] and PSO [5]. To test BFBBO, BBO, GBBO, PSO over the considered optimization test problems, following an experimental setting is adopted:
pMutation $=0.1$
$\alpha=0.9$

## 3. Result Comparison

To validate the performance of BFBBO it is also compared with BBO, GBBO, and PSO and the results are depicted in Table 2. The comparison is made based on the success rate (SR), average number of function evaluations (AFE) mean error (ME) and standard deviation (SD). The results reveal that BFBBO outperforms to the other respective algorithms in terms of accuracy, efficiency as well as reliability.

## 4. Statistical Analysis

In order to analyze convergence speed, AR is used which is represented as follows, based on the AFEs for the four other considered algorithms and BFBBO:
$\mathrm{AR}=\mathrm{AFEALG} /$ AFET FBBO
where $\mathrm{ALG} \epsilon(\mathrm{BBO}, \mathrm{GBBO}$ and PSO$)$ and $\mathrm{AR}>1$ means that BFBBO is speedy than the other considered algorithms. In order to examine the AR of the developed algorithm, as compared to the BBO, GBBO and PSO results of Table 2 are analysed and the AR's value is evaluated by using Equation 7. Table 3 shows a clear comparison among BFBBO and BBO, BFBBO and GBBO and BFBBO and PSO in terms of AR. It is clear from Table 3 that the CS of BFBBO is speedy among all the considered algorithms.

Table 2. Comparison of the results of test-functions, TPs: Test-Problems

| TPs | Algorithm | SD | ME | AFE | SR |
| :---: | :---: | :---: | :---: | :---: | :---: |
| f unc1 | BFBBO | 3.18E-02 | $3.49 \mathrm{E}-02$ | 243.33 | 30 |
|  | BBO | $2.11 \mathrm{E}-02$ | $6.66 \mathrm{E}-02$ | 311.67 | 30 |
|  | GBBO | $1.49 \mathrm{E}-05$ | $3.71 \mathrm{E}-05$ | 200000.00 | 0 |
|  | PSO | $1.88 \mathrm{E}-06$ | 7.70E-06 | 7360.00 | 30 |
| f unc2 | BFBBO | $2.63 \mathrm{E}-06$ | $4.05 \mathrm{E}-06$ | 4971.70 | 30 |
|  | BBO | $1.62 \mathrm{E}-06$ | $8.55 \mathrm{E}-06$ | 10546.67 | 30 |
|  | GBBO | $1.16 \mathrm{E}-06$ | $8.86 \mathrm{E}-06$ | 23301.67 | 30 |
|  | PSO | $1.24 \mathrm{E}-06$ | $8.49 \mathrm{E}-06$ | 6450.00 | 30 |
| f unc3 | BFBBO | $2.66 \mathrm{E}-06$ | $6.33 \mathrm{E}-06$ | 6836.70 | 30 |
|  | BBO | $1.52 \mathrm{E}-06$ | $8.91 \mathrm{E}-06$ | 22818.34 | 30 |
|  | GBBO | 8.46E-04 | $7.61 \mathrm{E}-01$ | 200000.00 | 0 |
|  | PSO | $2.82 \mathrm{E}-03$ | $7.60 \mathrm{E}-01$ | 200000.00 | 0 |
| f unc4 | BFBBO | 8.06E-02 | $2.89 \mathrm{E}+01$ | 50050.00 | 0 |
|  | BBO | 4.12E+02 | $1.88 \mathrm{E}+02$ | 200000.00 | 0 |
|  | GBBO | 4.12E+02 | $1.88 \mathrm{E}+02$ | 200000.00 | 0 |
|  | PSO | $1.48 \mathrm{E}+01$ | 2.18E+01 | 200000.00 | 0 |
| f unc5 | BFBBO | $1.00 \mathrm{E}+00$ | $3.00 \mathrm{E}-01$ | 14000.00 | 27 |
|  | BBO | 1.12E+01 | 3.93E+01 | 200000.00 | 0 |
|  | GBBO | $1.02 \mathrm{E}+01$ | $3.90 \mathrm{E}+01$ | 200000.00 | 0 |
|  | PSO | $1.02 \mathrm{E}+01$ | $3.90 \mathrm{E}+01$ | 200000.00 | 0 |
| f unc6 | BFBBO | 1.98E-06 | $7.68 \mathrm{E}-06$ | 10700.00 | 30 |
|  | BBO | 1.28E-06 | $8.61 \mathrm{E}-06$ | 60383.34 | 30 |
|  | GBBO | $5.25 \mathrm{E}-03$ | 2.67E-02 | 200000.00 | 0 |
|  | PSO | $8.26 \mathrm{E}-01$ | 7.79E-01 | 107183.34 | 15 |
|  | BFBBO | 3.16E-02 | 8.20E-03 | 15737.00 | 25 |

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| f unc7 | BBO | 2.90E-03 | $8.36 \mathrm{E}-03$ | 200000.00 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | GBBO | $2.90 \mathrm{E}-03$ | $8.36 \mathrm{E}-03$ | 200000.00 | 0 |
|  | PSO | $5.51 \mathrm{E}-04$ | $1.40 \mathrm{E}-04$ | 86393.34 | 19 |
| f unc8 | BFBBO | $5.21 \mathrm{E}-01$ | $6.51 \mathrm{E}+00$ | 50050.00 | 0 |
|  | BBO | $2.81 \mathrm{E}-01$ | $6.20 \mathrm{E}-01$ | 200000.00 | 0 |
|  | GBBO | $3.45 \mathrm{E}-01$ | $6.34 \mathrm{E}-01$ | 200000.00 | 0 |
|  | PSO | $9.42 \mathrm{E}-01$ | $1.69 \mathrm{E}+00$ | 200000.00 | 0 |
| f unc9 | BFBBO | 2.87E-06 | $6.05 \mathrm{E}-06$ | 5798.30 | 30 |
|  | BBO | 1.97E-01 | 2.36E-01 | 156776.67 | 7 |
|  | GBBO | 2.93E-01 | $8.77 \mathrm{E}-01$ | 200000.00 | 0 |
|  | PSO | $4.28 \mathrm{E}-01$ | $9.95 \mathrm{E}-01$ | 200000.00 | 0 |
| f unc10 | BFBBO | $2.23 \mathrm{E}-06$ | $5.74 \mathrm{E}-06$ | 4950.00 | 30 |
|  | BBO | 1.88E-06 | $9.06 \mathrm{E}-06$ | 10276.67 | 30 |
|  | GBBO | $6.25 \mathrm{E}-07$ | $9.50 \mathrm{E}-06$ | 65376.67 | 30 |
|  | PSO | $2.50 \mathrm{E}-06$ | 7.47E-06 | 7320.00 | 30 |
| f unc11 | BFBBO | 2.70E-03 | $5.00 \mathrm{E}-03$ | 8206.70 | 30 |
|  | BBO | $9.25 \mathrm{E}-01$ | $1.11 \mathrm{E}+00$ | 200000.00 | 0 |
|  | GBBO | $1.06 \mathrm{E}-03$ | $1.00 \mathrm{E}-02$ | 137120.00 | 27 |
|  | PSO | 3.63E-04 | $9.50 \mathrm{E}-03$ | 52376.67 | 30 |
| f unc12 | BFBBO | $2.07 \mathrm{E}-06$ | $6.94 \mathrm{E}-06$ | 10125.00 | 30 |
|  | BBO | $2.22 \mathrm{E}-06$ | $8.01 \mathrm{E}-06$ | 53448.34 | 30 |
|  | GBBO | $2.83 \mathrm{E}+00$ | $9.42 \mathrm{E}+00$ | 200000.00 | 0 |
|  | PSO | 2.02E-06 | $8.42 \mathrm{E}-06$ | 12746.67 | 30 |
| f unc13 | BFBBO | 2.44E-06 | $6.00 \mathrm{E}-06$ | 6320.00 | 30 |
|  | BBO | $7.69 \mathrm{E}-07$ | $9.23 \mathrm{E}-06$ | 21733.34 | 30 |
|  | GBBO | $5.02 \mathrm{E}-06$ | $1.79 \mathrm{E}-05$ | 199925.00 | 1 |
|  | PSO | $1.75 \mathrm{E}-06$ | 8.14E-06 | 7933.34 | 30 |
| f unc14 | BFBBO | $1.64 \mathrm{E}-06$ | 7.80E-06 | 9863.30 | 30 |
|  | BBO | $9.51 \mathrm{E}-07$ | $9.09 \mathrm{E}-06$ | 48673.34 | 30 |
|  | GBBO | $6.77 \mathrm{E}-03$ | 3.32E-02 | 200000.00 | 0 |
|  | PSO | $4.47 \mathrm{E}-02$ | $1.32 \mathrm{E}-02$ | 158420.00 | 7 |
| f unc15 | BFBBO | 2.79E-02 | $1.31 \mathrm{E}-01$ | 8311.70 | 29 |
|  | BBO | 1.12E-01 | $6.73 \mathrm{E}-01$ | 200000.00 | 0 |
|  | GBBO | $5.39 \mathrm{E}-02$ | $4.10 \mathrm{E}-01$ | 200000.00 | 0 |
|  | PSO | $6.53 \mathrm{E}-02$ | 3.20E-01 | 175233.34 | 4 |
| f unc16 | BFBBO | 2.26E-06 | $6.52 \mathrm{E}-06$ | 6960.00 | 30 |
|  | BBO | $1.34 \mathrm{E}-06$ | $8.88 \mathrm{E}-06$ | 23606.67 | 30 |
|  | GBBO | $2.41 \mathrm{E}-04$ | $5.22 \mathrm{E}-04$ | 20000.00 | 0 |
|  | PSO | $1.45 \mathrm{E}-06$ | $8.28 \mathrm{E}-06$ | 8456.67 | 30 |
| f unc17 | BFBBO | 1.46E-01 | $1.31 \mathrm{E}-01$ | 24363.00 | 21 |
|  | BBO | $7.39 \mathrm{E}-02$ | $2.00 \mathrm{E}-01$ | 198528.34 | 1 |
|  | GBBO | $3.34 \mathrm{E}-01$ | $9.74 \mathrm{E}-01$ | 200000.00 | 0 |
|  | PSO | $5.99 \mathrm{E}-01$ | $1.36 \mathrm{E}+00$ | 200000.00 | 0 |
| f unc18 | BFBBO | 2.94E-06 | $5.51 \mathrm{E}-06$ | 3028.30 | 30 |
|  | BBO | $5.66 \mathrm{E}-06$ | $8.56 \mathrm{E}-06$ | 42401.67 | 27 |
|  | GBBO | $2.26 \mathrm{E}-06$ | $7.49 \mathrm{E}-06$ | 4306.67 | 30 |
|  | PSO | $2.04 \mathrm{E}-06$ | $7.49 \mathrm{E}-06$ | 5243.34 | 30 |
| f unc19 | BFBBO | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | 3780.00 | 30 |
|  | BBO | $0.00 \mathrm{E}+00$ | 0.00E+00 | 4455.00 | 30 |
|  | GBBO | $0.00 \mathrm{E}+00$ | 0.00E+00 | 5521.67 | 30 |
|  | PSO | $0.00 \mathrm{E}+00$ | 0.00E+00 | 55211.67 | 2 |
|  | BFBBO | 2.67E-06 | $5.87 \mathrm{E}-06$ | 7785.00 | 30 |

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| f unc20 | BBO | $1.58 \mathrm{E}-06$ | $8.88 \mathrm{E}-06$ <br>  <br>  <br>  <br>  | GBBO | 2.49285 .00 | 30 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PSO | $2.49 \mathrm{E}-03$ | $5.42 \mathrm{E}-03$ | 200000.00 | 0 |  |  |
| $5.42 \mathrm{E}-03$ | 200000.00 | 0 |  |  |  |  |

Table 3. Test Problems: TPs, Acceleration Rate (AR) of BFBBO compare to the basic BBO, GBBO and PSO

| TPs | BBO | GBBO | PSO |
| :--- | :--- | :--- | :--- |
| $f$ unc1 | 1.28082 | 821.91792 | 30.24658 |
| $f$ unc2 | 2.12134 | 4.68686 | 1.29734 |
| $f$ unc3 | 3.33762 | 29.25388 | 29.25388 |
| $f$ unc4 | 3.99600 | 3.99600 | 3.99600 |
| $f$ unc5 | 14.28571 | 14.28571 | 14.28571 |
| $f$ unc6 | 5.64330 | 18.69159 | 10.01713 |
| $f$ unc7 | 12.70890 | 12.70890 | 5.48982 |
| $f$ unc8 | 3.99600 | 3.99600 | 3.99600 |
| $f$ unc9 | 27.03838 | 34.49287 | 34.49287 |
| $f$ unc10 | 2.07609 | 13.20741 | 1.47879 |
| $f$ unc11 | 24.37033 | 16.70830 | 6.38218 |
| $f$ unc12 | 5.27885 | 19.75309 | 1.25893 |
| $f$ unc13 | 3.43882 | 31.63370 | 1.25527 |
| $f$ unc14 | 4.93479 | 20.27719 | 16.06156 |
| $f$ unc15 | 24.06247 | 24.06247 | 21.08273 |
| $f$ unc16 | 3.39176 | 28.73563 | 1.21504 |
| $f$ unc17 | 8.14876 | 8.20917 | 8.20917 |
| $f$ unc18 | 14.00181 | 1.42214 | 1.73144 |
| $f$ unc19 | 1.17857 | 1.46076 | 14.60626 |
| $f$ unc20 | 3.99600 | 3.99600 | 3.99600 |

The empirical-distribution of data graphically depicted efficiently by comparing the considered-algorithms in form of consolidated-performance which is carried out by boxplot-analysis of BFBBO For BFBBO, BBO, GBBO and PSO's boxplots are exhibited in Figure 4. The results reveals that BFBBO's medians and interquartile range are relatively very less.

Another boxplot for BFBBO, BBO [2], GBBO [4] and PSO [5] also generated on the basis of success rate evaluation in Figure \ref\{fig:boxplot2\}. The results reveals that interquartile range and medians of BFBBO are relatively high. The algorithms are also assessed by Mann-Whitney U rank (MWUR) sum test. MWUR sum test is applied to AFEs and for all the considered algorithms the experiment is performed at $5 \%$ significance level ( $\alpha=0.005$ ). The result for 100 runs are presented in Table 4. It is clear from the Table 4 that this strategy performs better as compared to the other respected algorithms.


Fig. 4: Boxplots Graph for AFEs


Fig. 5: Boxplots Graph for SRs
Table 4. Test Problems: TPs, Mann Whitney U rank (MWUR)sum test ('+’'indicates BFBBO is better, ‘-’ indicates BFBBO is worst and ' $=$ ' indicates that there no noticeable difference)

| TPs | $\begin{aligned} & \mathrm{BFBBO} \\ & \mathrm{BBO} \end{aligned}$ | $\begin{gathered} \hline \text { vs } \mathrm{BFBBO} \\ \text { GBBO } \end{gathered}$ | vsBFBBO vs PSO |
| :---: | :---: | :---: | :---: |
| f unc1 | + ve | + ve | + ve |
| f unc2 | + ve | + ve | + ve |
| f unc3 | + ve | + ve | + ve |
| f unc4 | + ve | + ve | + ve |
| f unc5 | + ve | + ve | + ve |
| f unc6 | + ve | + ve | + ve |
| f unc7 | + ve | + ve | + ve |
| f unc8 | + ve | + ve | + ve |
| f unc9 | + ve | + ve | + ve |
| f unc10 | + ve | + ve | + ve |
| f unc11 | + ve | + ve | + ve |
| f unc12 | + ve | + ve | + ve |
| f unc13 | + ve | + ve | + ve |
| f unc14 | + ve | + ve | + ve |
| f unc15 | + ve | + ve | + ve |
| f unc16 | + ve | + ve | + ve |
| f unc17 | + ve | + ve | + ve |
| f unc18 | + ve | + ve | + ve |

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| f unc19 | + ve | + ve | + ve |
| :---: | :---: | :---: | :---: |
| f unc20 | + ve | + ve | + ve |
| Total number sign | 20 | 20 | 20 |

## V. CONCLUSION

In this paper, to reduce the step size of BBO, butterfly curve inspired local search strategy has been incorporated with BBO. The proposed strategy is epithet as Butterfly Curve based BBO (BFBBO) algorithm. The developed algorithm is tested over 20 well known benchmark test functions through various statistical analysis and found that for solving the continuous optimization-problems BFBBO may be a effective choice.

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